Exercise 2

Solve the differential equation.

$$y'' - 6y' + 9y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Plug these formulas into the ODE.

$$r^2 e^{rx} - 6(re^{rx}) + 9(e^{rx}) = 0$$

Divide both sides by e^{rx}.

$$r^2 - 6r + 9 = 0$$

Solve for r.

$$(r-3)^2 = 0$$

$$r = \{3\}$$

Two solutions to the ODE are e^{3x} and xe^{3x}. By the principle of superposition, then,

$$y(x) = C_1 e^{3x} + C_2 x e^{3x},$$

where C_1 and C_2 are arbitrary constants.